EE-565 – W2
DC MACHINE MODELING

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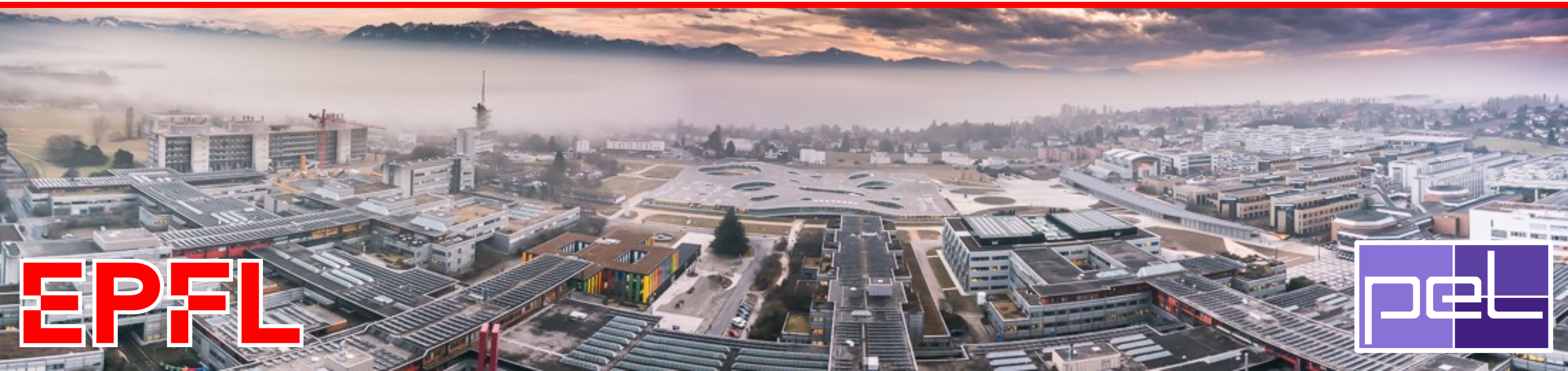


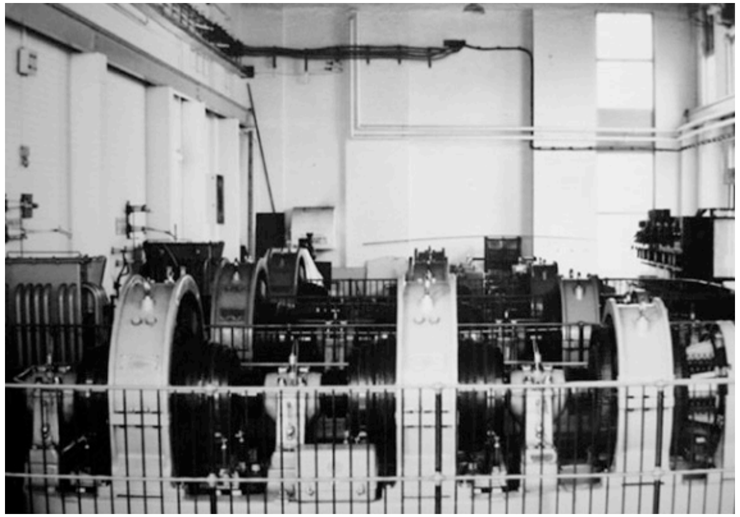
DC MACHINE FUNDAMENTALS

Operating principles, constructions details

HISTORY

In the early days of electrification, DC electrical machines were developed first

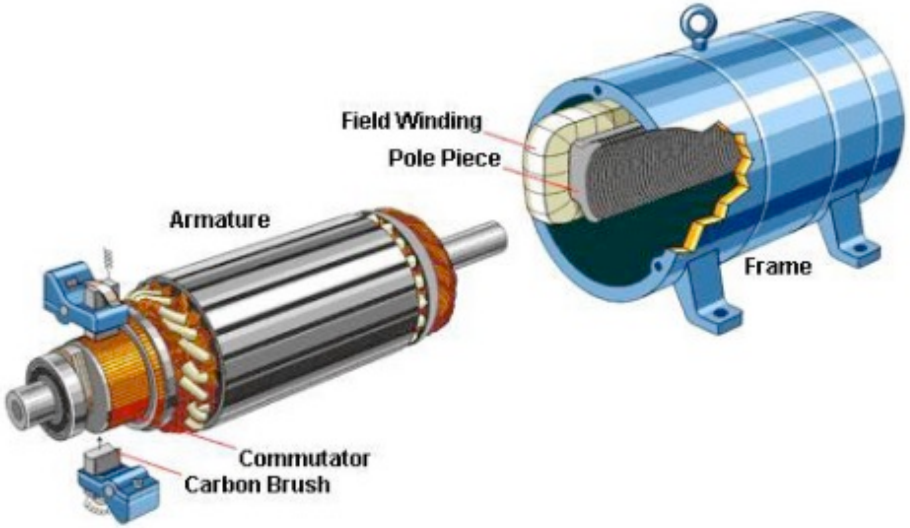
- production, transmission and application of electrical energy were based on DC voltages and currents
- output voltage of DC generators was limited to several hundreds of volts
- high power transmission was characterized with high currents and thus high transmission losses
- today's power system is based on AC voltages and currents, and thus AC machines are much more popular
- nevertheless, DC machines were the first machines widely used



DC MACHINES

A DC machine consist of:

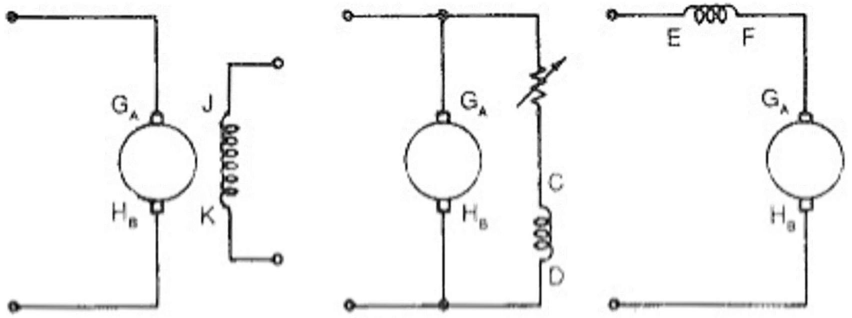
- stator magnetic circuit
- rotor magnetic circuit
- stator may have stator winding (**excitation winding**) or permanent magnets
- rotor has rotor winding, called also **armature winding**
- both stator and rotor electrical access terminals are supplied by DC current
- stator and rotor fluxes are created due to respective currents flowing in winding



DC MACHINE TYPES

Different connections can be used for Armature and Excitation windings:

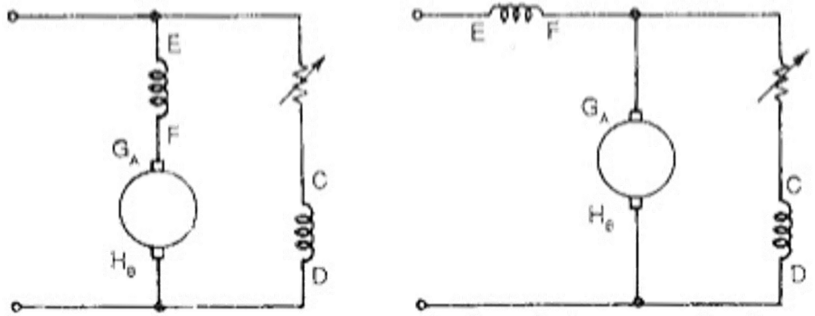
- **Separately Excited DC Machines**
  - The two windings are supplied independently from one another
  - The excitation does not change with the rotor (armature) current
  - Is the **mostly used for variable speed operation**
- **Series Connected DC Machines**
  - The two windings are connected in series
  - The excitation is proportional to the rotor (armature) current
  - Allows also use with AC supply (**Universal Motors**)
- **Parallel Connected DC Machines**
  - The two windings are connected in parallel
  - The excitation depends on the supplying voltage
- **Compound DC Machines**
  - The two windings are connected partially in series and partially in parallel
  - The series and parallel field windings can be connected differently



(a) Separately excited dc motor.

(b) Shunt motor

(c) Series motor



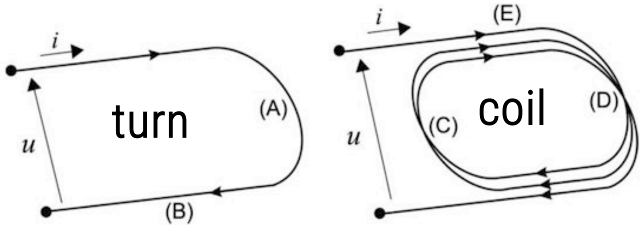
(d) Long shunt compound motor

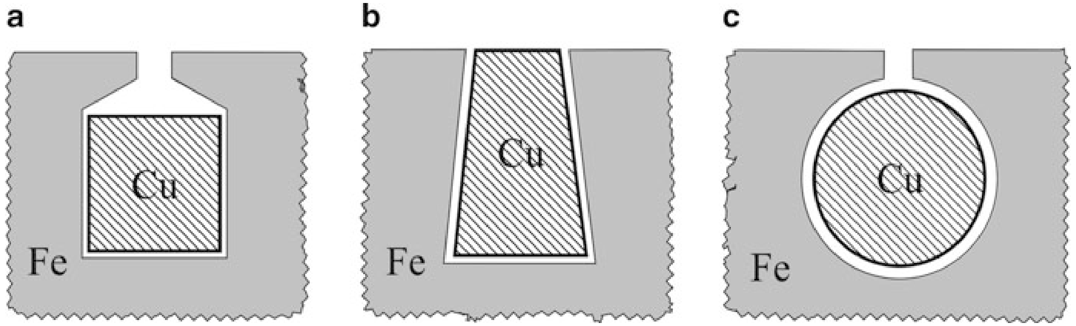
(e) Short shunt compound motor

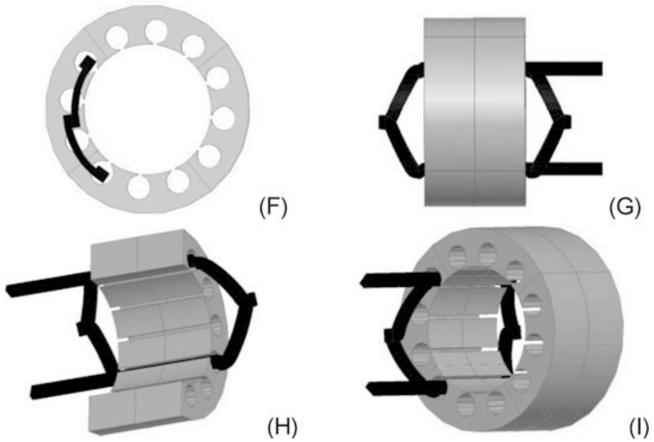
SLOTS IN MAGNETIC CIRCUITS

Magnetic circuits of the stator and rotor are made of iron sheets

- iron sheets (SiFe) are insulated from each other and have **slots** to host windings
- windings are also insulated from each other
- the slots can be of different cross section, depending on the design needs
- the slots usually host more than one conductor (not necessarily of the same winding)
- **Tooth** is the part of the magnetic circuit between neighboring slots
- one **turn** is obtained by a series connection of conductors placed in different slots
- several turns may reside in the same pair of slots, creating a **coil**
- flux of one turn Φ is equal to the flux through the contour defined by its conductors
- flux of a coil having N turns is equal to: $N\Phi$



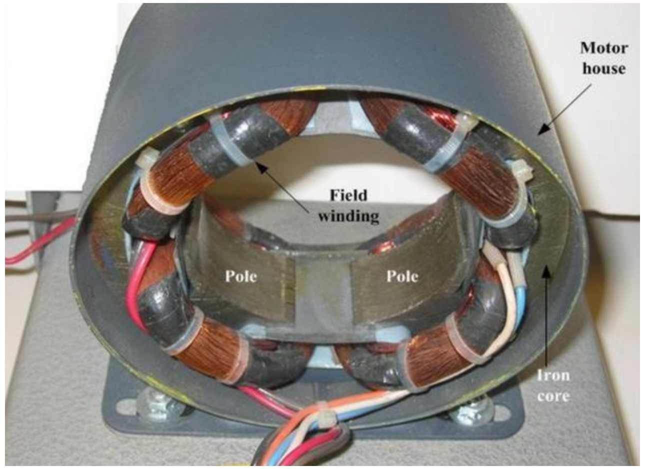


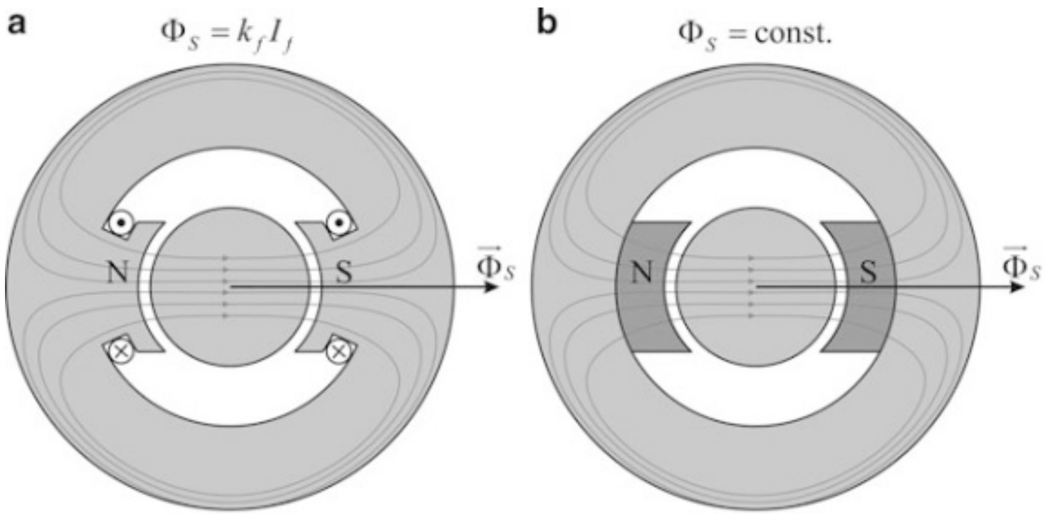


STATOR

The excitation flux on the stator can be generated by:

- DC currents in a **Stator Winding** (**Field** or **Excitation Winding**)
  - Can be controlled
- **Permanent Magnets** built in the stator magnetic circuit
  - Cannot be controlled

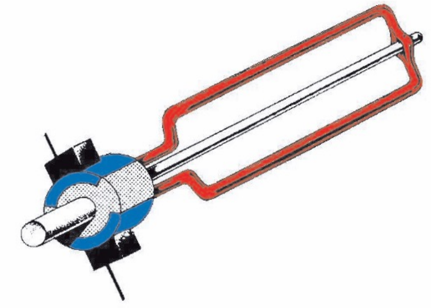


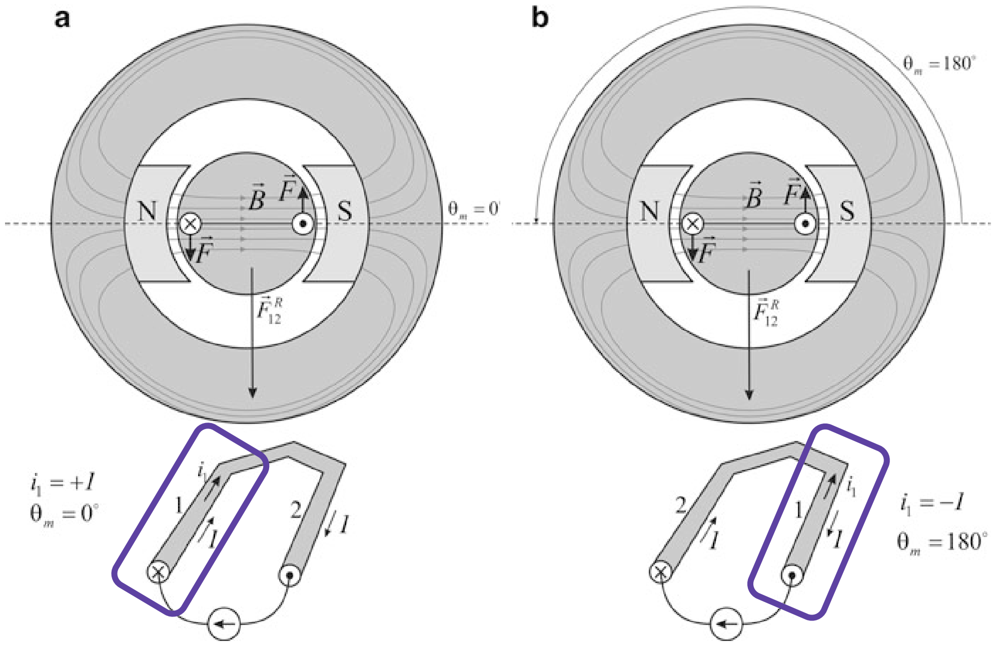


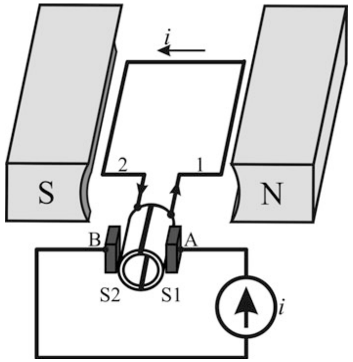
CURRENTS IN THE ROTOR CONDUCTORS

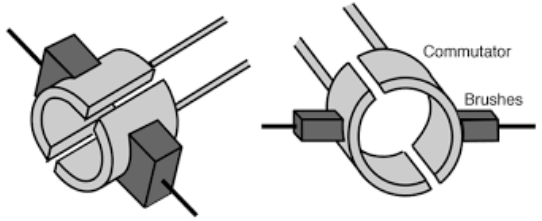
Rotor conductors are axially mounted inside the stator magnetic circuits:

- ► rotor conductors with currents interact with magnetic induction of the stator excitation field
- ► a couple of **Forces** create mechanical **Torque** that sets rotor in motion
- ► to maintain torque generation, rotor current direction should not change under the stator pole
- ► **rotor current direction inside the contour must periodically change** – function of the rotor position
- ► rotor winding currents are therefore AC - achieved through **mechanical commutator**







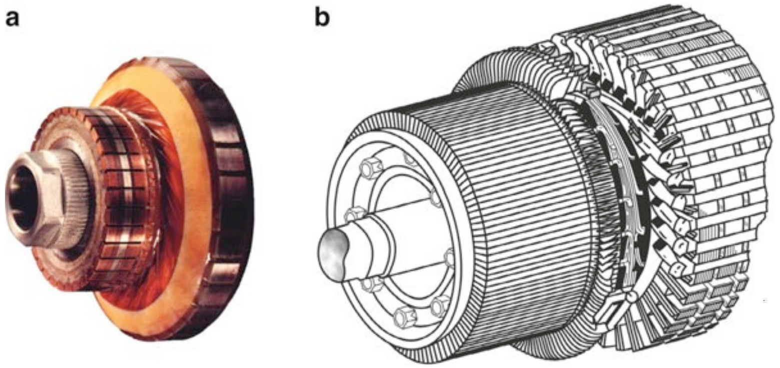


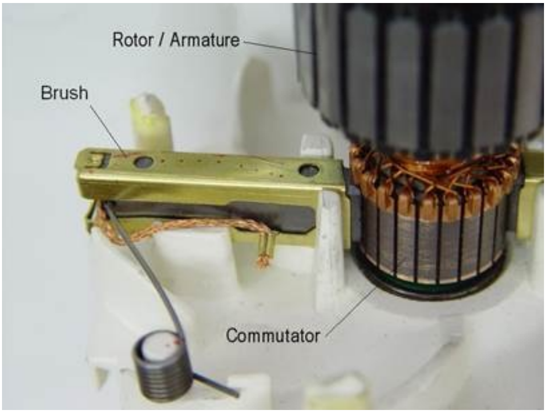
MECHANICAL COMMUTATOR

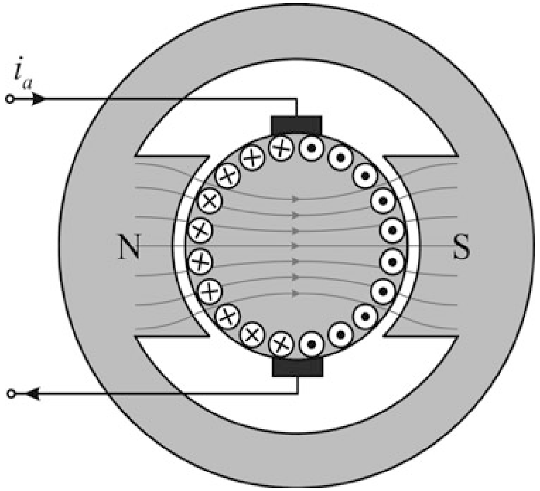
A **mechanical commutator** is located **between rotor electrical access and rotor conductors**

- **Collector** – rotating part made of collector segments where rotor conductors are terminated
- **Brushes** – fixed and bring external electrical currents to the collector ring

- The mechanical commutator converts DC currents from the power supply to periodic AC currents inside the rotor
- The frequency of rotor currents is determined by the speed of rotation
- There are issues associated with commutator are related to mechanical wear, particles, dust, sparks







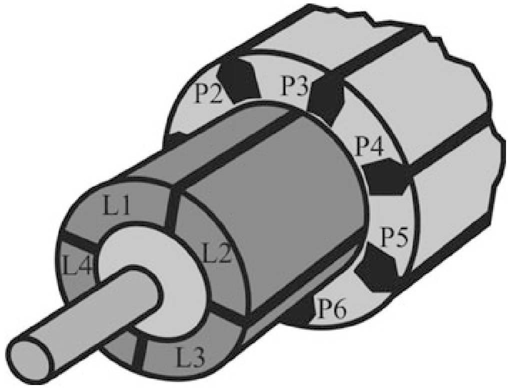
ROTOR WINDING AND COMMUTATOR

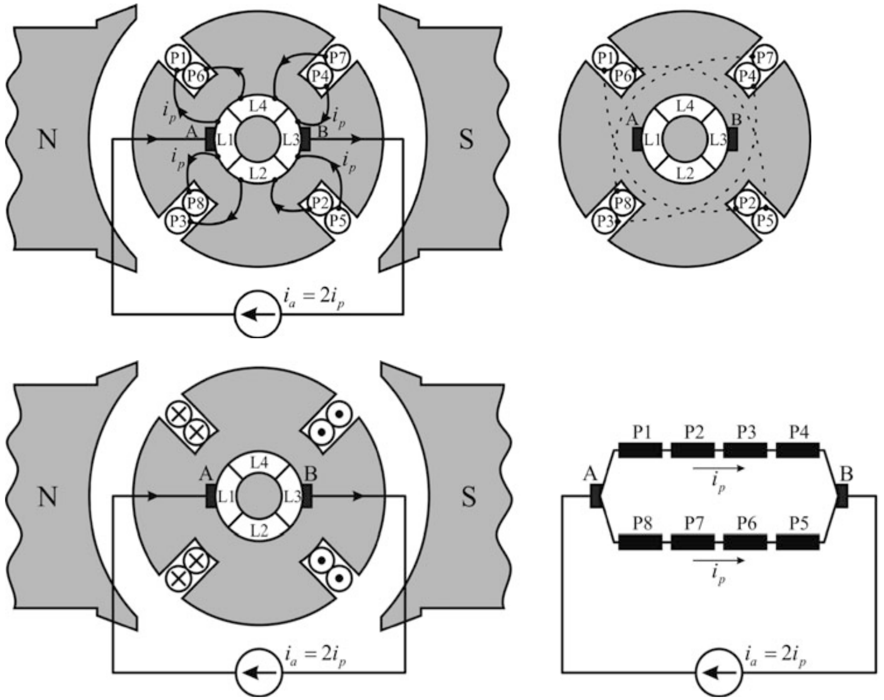
Usually, there is a large number of rotor conductors and collector segments

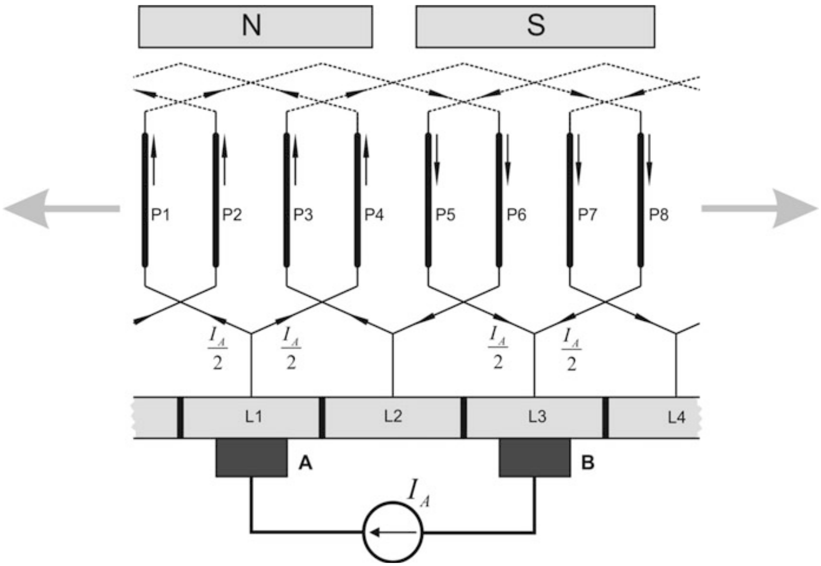
Example (illustration only)

- 4 rotor slots, 4 collector segments, 8 conductors
- 2 conductors per slot
- 4 turns: (P1-P2), (P3-P4), (P5-P6), (P7-P8)
- Brush A in contact with L1
- Brush B in contact with L3

- The source current is split in two parallel paths
- The direction of the currents under the poles stays the same



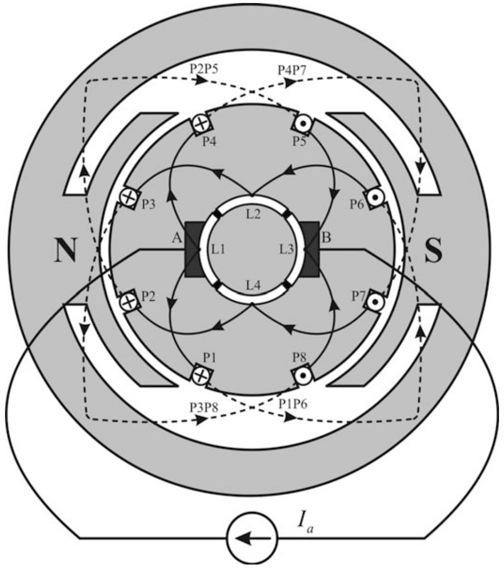


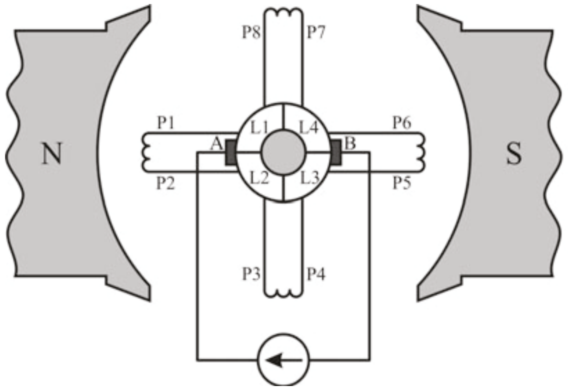


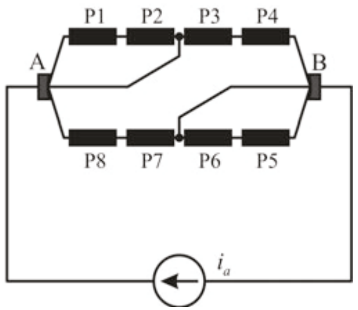
PROBLEMS WITH COMMUTATION

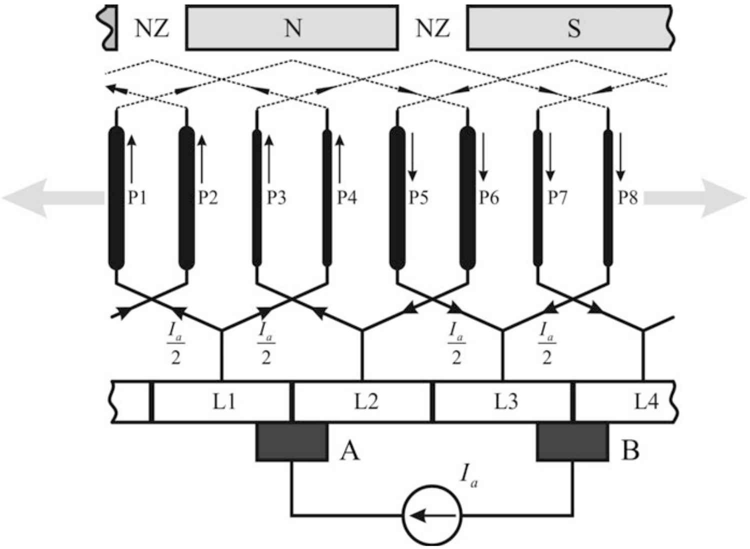
During the commutation, commutator segments are short circuited

- **Change of current direction** happens in those **conductors that are short-circuited**
- Short circuit current is limited **only by the impedance of the turn**
- **Excessive overheating** of the brushes can result in an electric arc, increasing the wear
- Fue to rotation, **sparking,** arcing ionized particles may cause circulating arcing
- During commutation, it is important to keep induced emf in short-circuted turns close to zero
- **The neutral zone (NZ) should be sufficiently wide**





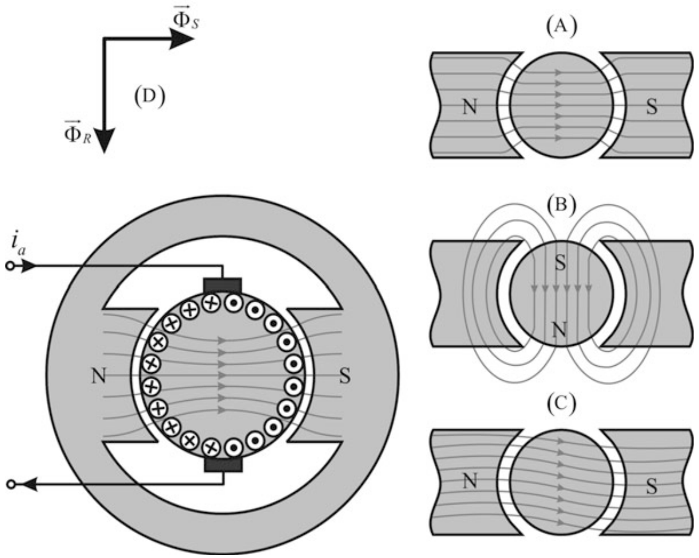




ROTOR MAGNETIC FIELD

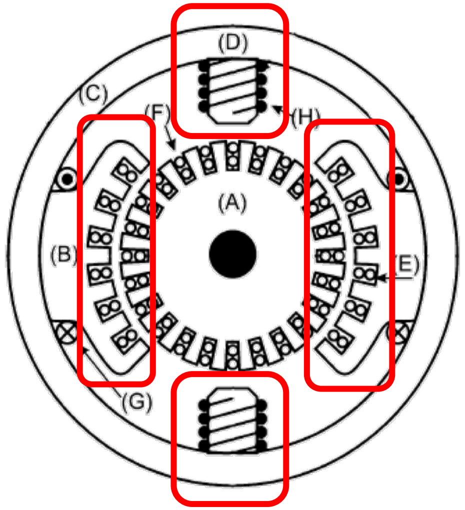
Rotor currents create rotor flux that has poles in the neutral zone

- the MMF force and flux created by the rotor currents are called **armature reaction** (to the stator excitation)
- rotor flux lines pass through regions of very low permeability and very high magnetic resistance
- magnetic induction in the neutral zone (NZ) is rather low, but still **it can affect commutation**



To **reduce magnetic induction in the neutral zone**, DC machines may have

- **Compensation windings**
- **Auxiliary Poles windings**

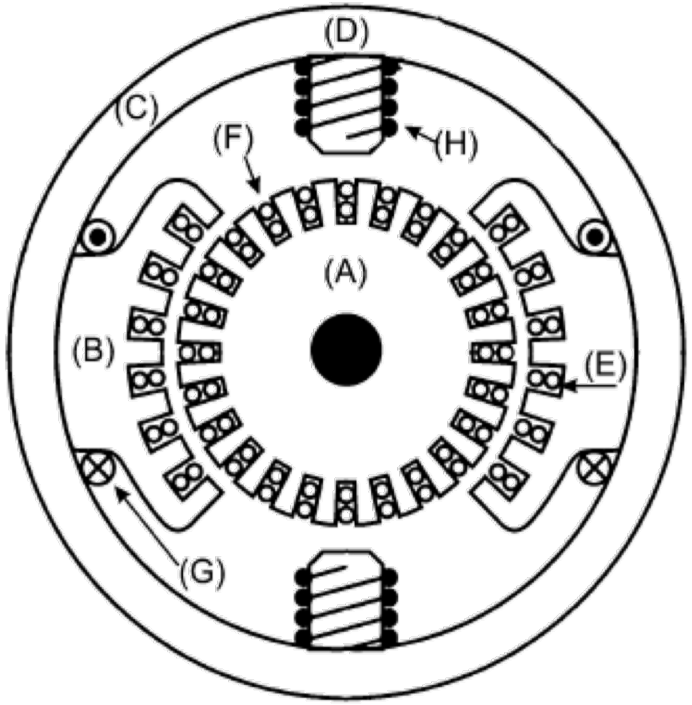


Note: the compensation windings and auxiliary poles windings are physically located on the stator, but are supplied by the same current of the armature (rotor)

CURRENT AND MAGNETIC CIRCUITS

In summary, a DC machine has:

- **Rotor magnetic circuit – A**
  subject to variable magnetic field, that produces hysteresis and eddy current losses
  made of iron laminated sheets
- **Main poles – B**
  Main part of the stator magnetic circuit, with low air-gap
- **Yoke – C**
  magnetic circuit subject to constant flux and magnetic induction
  it does not have to be laminated and can be made of solid iron
- **Auxiliary poles – D**
  have larger air gap than main poles, to increase magnetic resistance
- **Compensation winding – E**
  supplied by the rotor (armature) current of the opposite direction
  cancel the MMF of all rotor inductors located under the main stator poles
- **Rotor current circuit – F**
  connected to the commutator
- **Stator current circuit – G**
  excitation winding
- **Auxiliary poles winding – H**
  supplied by the rotor (armature) current, and is made such to compensate rotor MMF not compensated by compensation winding



DC MACHINE MODELING

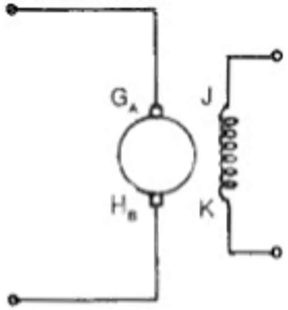
Mathematical model of the machine

DIRECT AND QUADRATURE AXIS

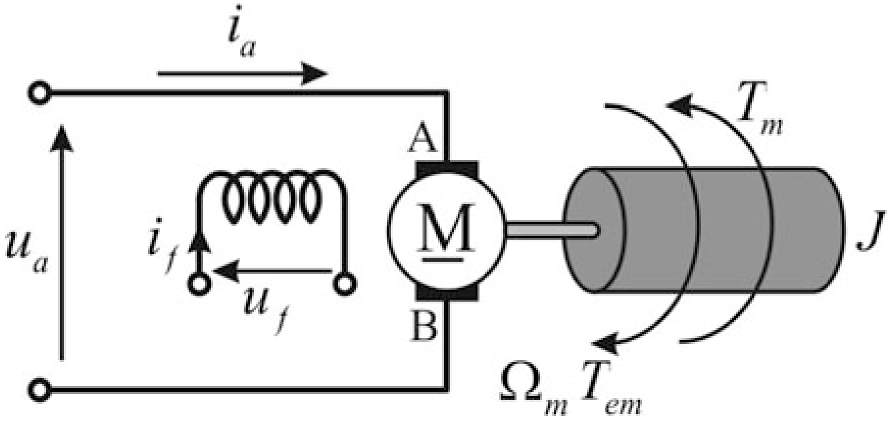
Magnetic axes of rotor and stator are perpendicular by machine design

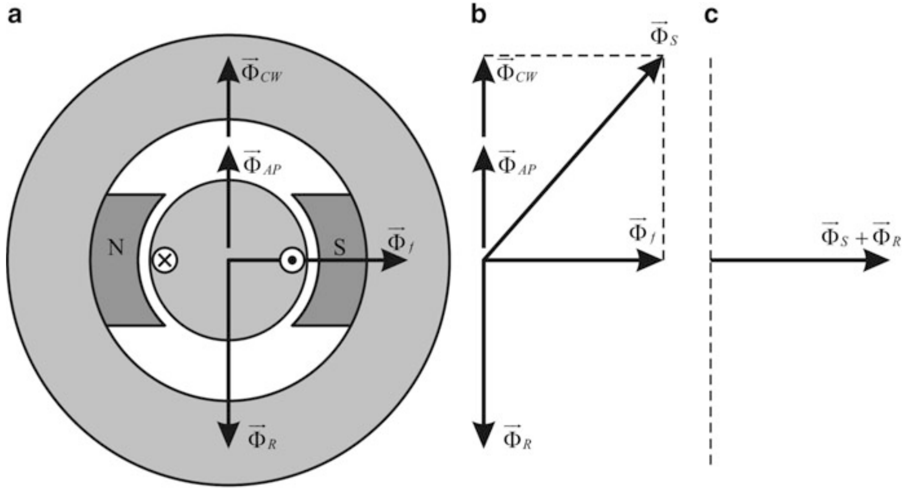
- **Direct axis** (horizontal)
  corresponds to the excitation flux and is determined by the position of main poles
- **Quadrature axis** (vertical)
  corresponds to the armature reaction

- the stator auxiliary poles and compensating winding act along the quadrature axis
- the stator has three fluxes Φ_f; Φ_{CW}; Φ_{AP} creating resulting stator flux Φ_S
- the rotor has one flux Φ_R
- for a full compensation, the fluxes Φ_{CW} and Φ_{AP} cancel the flux Φ_R, and the resulting flux in quadrature axis is zero
- resultant flux along the direct axis is equal to the excitation flux Φ_f



(a) Separately excited dc motor.





EXCITATION WINDING MODEL

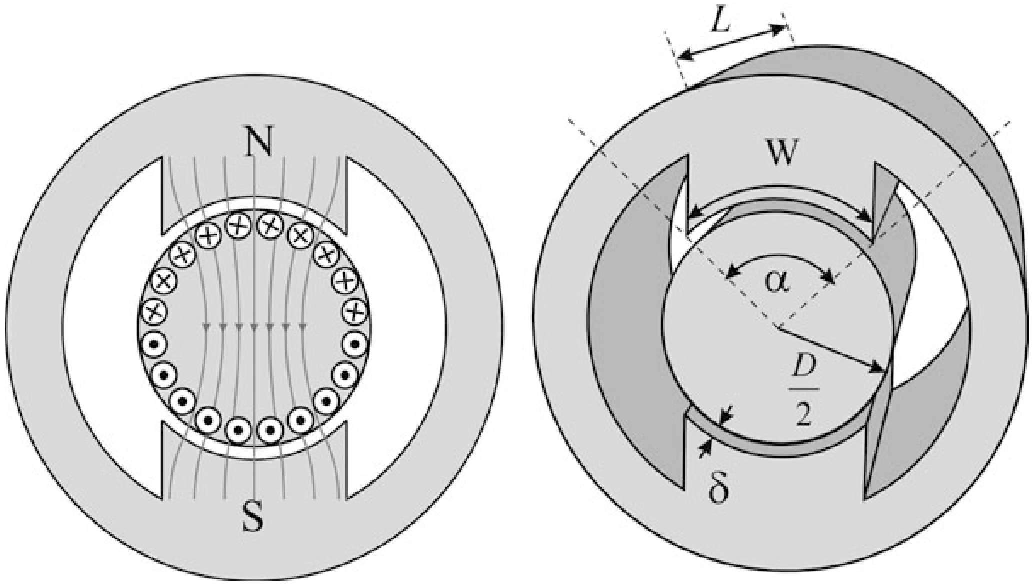
The EMF can be expressed in terms of rotor speed and excitation flux:

- The magnetic induction due to excitation winding, inside the air gap under main poles is:

$$B_f = \overbrace{\mu_0}^{\text{Magnetic Permeability}} \underbrace{H_f}_{\text{Magnetic field in the air-gap}} = \mu_0 \frac{\overbrace{N_f}^{\text{Turns in the excitation winding}} \overbrace{I_f}^{\text{Current in the excitation winding}}}{\underbrace{2\delta}_{\text{Air-gap length}}}$$

- The excitation flux is equal to the product of the magnetic induction and surface area:

$$\Phi_f = \overbrace{S}^{\text{Surface Area}} \cdot B_f = \mu_0 \frac{\overbrace{L}^{\text{Axial length}} \overbrace{W}^{\text{Width}} N_f I_f}{2\delta}$$



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- The magnetic induction due to excitation winding, inside the air gap under main poles is:

$$B_f = \mu_0 H_f = \mu_0 \frac{N_f I_f}{2\delta}$$

- The excitation flux is equal to the product of the magnetic induction and surface area:

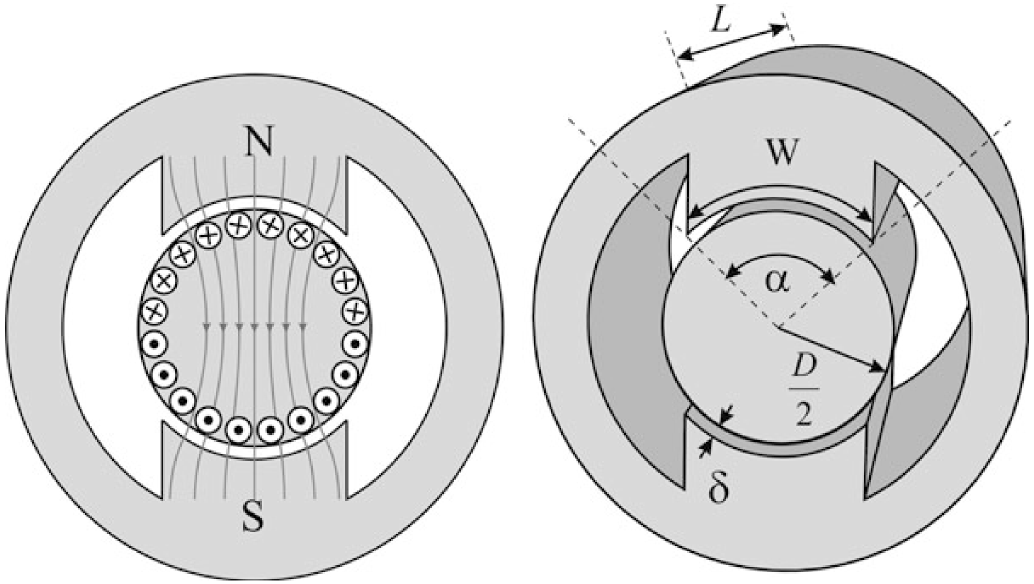
$$\Phi_f = S \cdot B_f = \mu_0 \frac{L\, W\, N_f I_f}{2\delta}$$

- The magnetic resistance along the excitation path is

$$R_\mu = \frac{F_f}{\Phi_f} = \frac{2\delta}{\mu_0\, L\, W}$$

- The inductance of the excitation winding is:

$$L_f = \frac{\Psi_f}{I_f} = \frac{N_f \Phi_f}{I_f} = \mu_0 \frac{L W N_f^2}{2\delta} = \frac{N_f^2}{R_\mu}$$

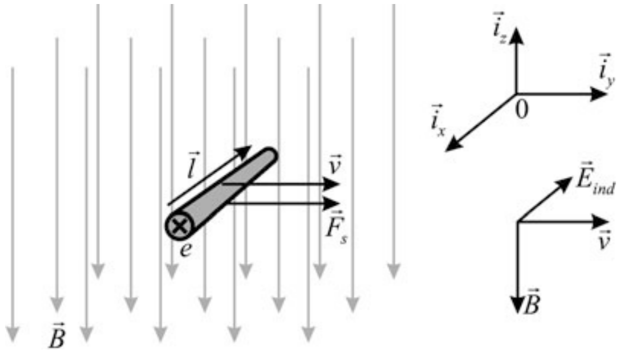


ELECTROMOTIVE FORCE CALCULATION

The EMF in a moving conductor can be found from its speed and the magnetic field:

$$\underbrace{\vec{E}_{ind}}_{\text{Electric Field in moving conductors}} = \overbrace{\vec{v}}^{\text{Conductor Speed}} \times \underbrace{\vec{B}}_{\text{Applied Magnetic Flux Density field}}$$

$$\underbrace{E}_{\text{Electromotive Force}} = \overbrace{\vec{l}}^{\text{Directed conductor length}} \cdot \underbrace{\vec{E}_{ind}}_{\text{Electric Field}} = \vec{l} \cdot (\vec{v} \times \vec{B})$$

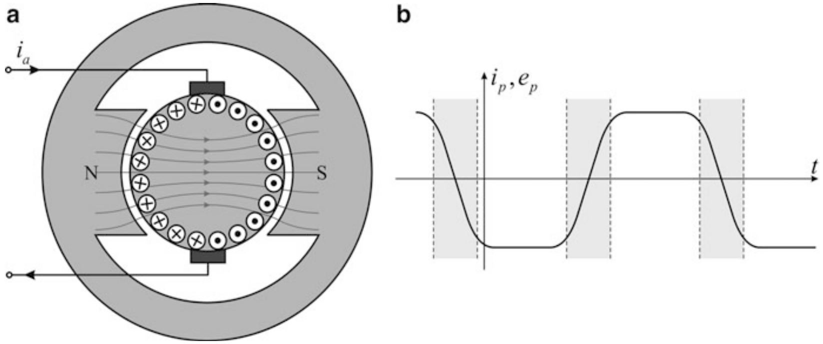


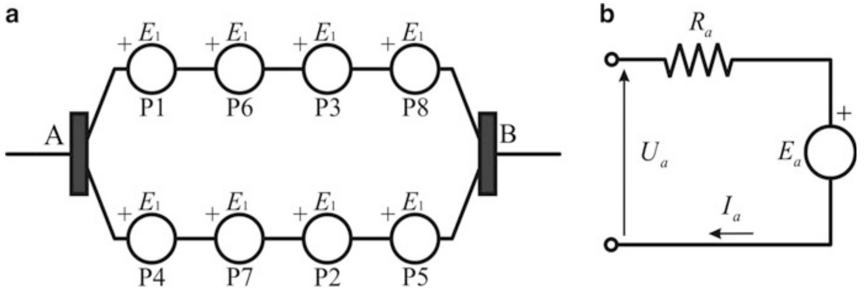
The EMF can be expressed in terms of rotor speed and excitation flux:

- In each of the rotor conductors passing under the main poles of the stator, there is induced EMF as:

$$E_k = \underbrace{\pm}_{\text{N/S Pole}} \overbrace{r}^{\text{Rotor radius}} \cdot \underbrace{\omega_m}_{\text{Rotor speed}} \cdot \overbrace{B_f}^{\text{Excitation field}} \cdot \underbrace{l}_{\text{Axial length}}$$

(Note: the induced emf in each conductor is AC by nature)





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- In each of the rotor conductors passing under the main poles of the stator, there is induced EMF as:

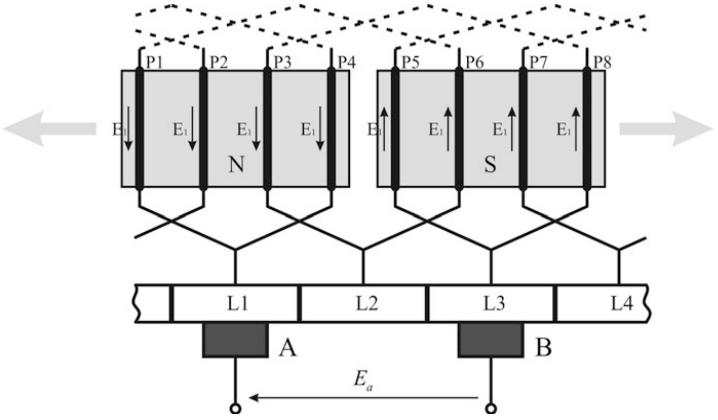
$$E_k = \pm\, r \cdot \omega_m \cdot B_f \cdot l$$

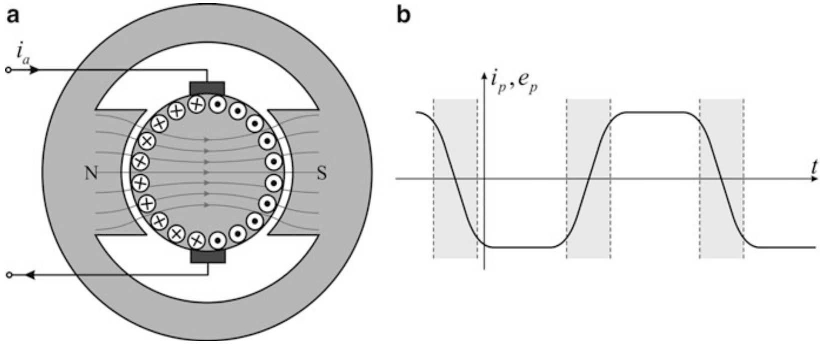
- The total induced emf in the armature (rotor) winding is the composition of the contribution of different conductors:

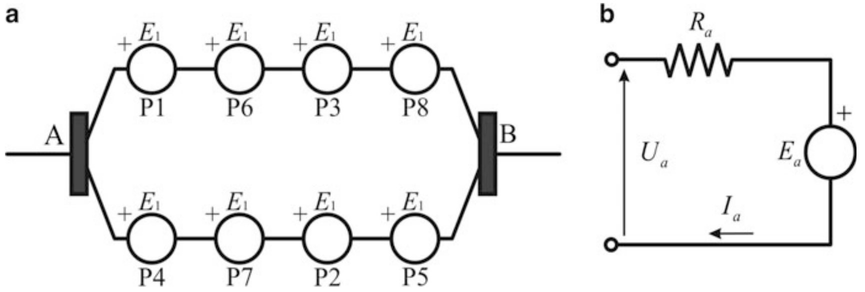
$$E_a = \sum_k E_k = \underbrace{\left(N_R \, \frac{W}{2\pi r}\right)}_{\text{Number of rotor conductors under one pole}} E_k = \left(\frac{N_R W}{2\pi r}\right) B_f \, l \, r \, \omega_m$$

(Note: the induced armature EMF is DC by re-combination from commutator)

The commutator works as a mechanical DC-AC converter







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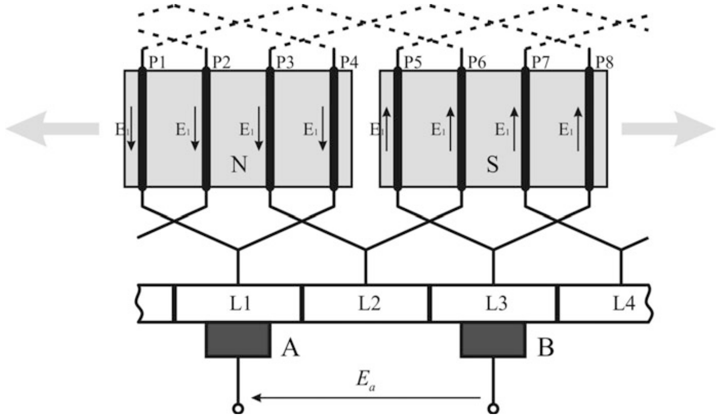
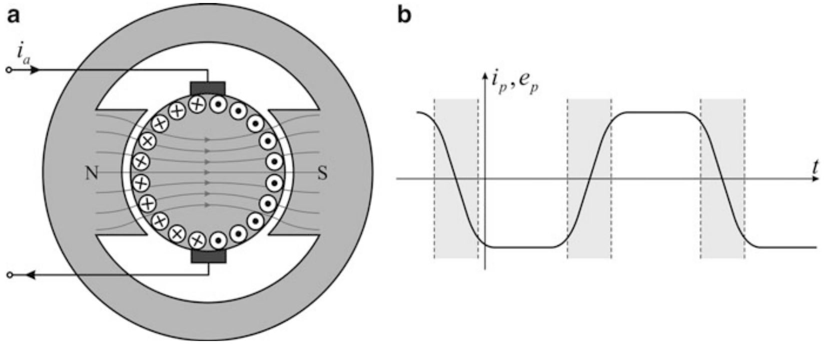
$$E_a = \sum_k E_k = \left(N_R \,\frac{W}{2\pi r}\right) E_k = \left(\frac{N_R W}{2\pi r}\right) B_f \, l \, r \, \omega_m$$

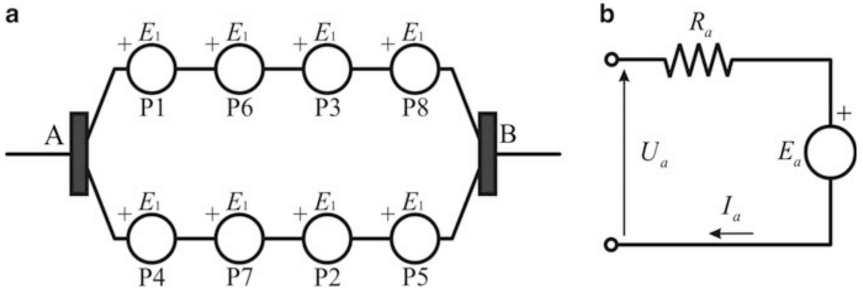
$$= \left(\frac{N_R}{2\pi}\right) \cdot (l \, W \, B_f) \cdot \omega_m = \underbrace{k_e}_{\text{EMF Coefficient}} \Phi_f \, \omega_m$$

(Note: the induced armature EMF is DC by re-combination from commutator)

The commutator works as a mechanical DC-AC converter

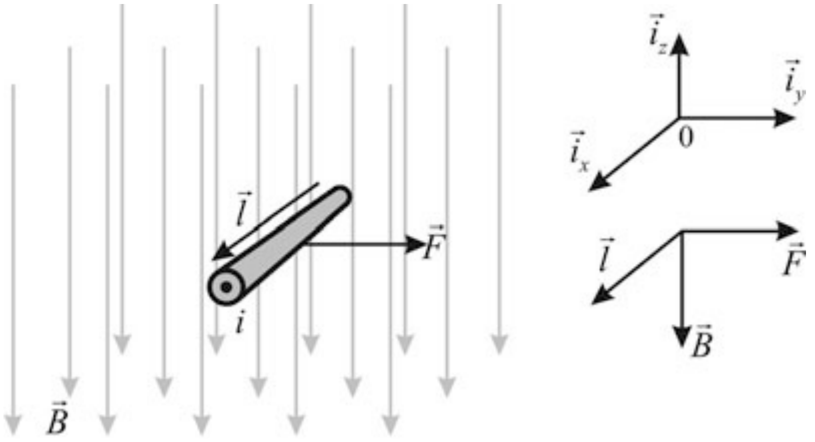






TORQUE GENERATION

Electromagnetic torque is related to the excitation flux and armature current



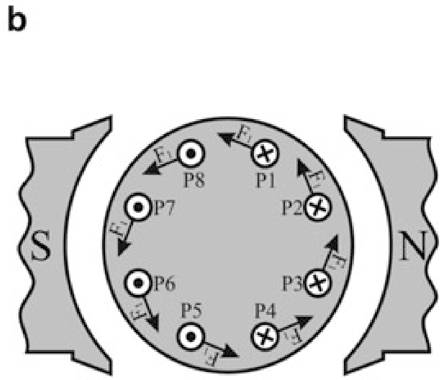
- in the zones below the poles, the vector product of radial component of magnetic induction and coaxially directed current gives tangential forces F

$$\underbrace{\vec{F}_k}_{\text{Force}} = \overbrace{i_k}^{\text{Conductor Current}} \cdot (\underbrace{\vec{l}}_{\text{Current oriented direction}} \times \overbrace{\vec{B}_f}^{\text{Air-gap Field}})$$

(Lorentz force)

- Each individual force contributes to the electromagnetic torque

$$\underbrace{T_{em,k}}_{\text{Torque of one conductor}} = |\overbrace{\vec{r}_k}^{\text{Arm for torque calculation}} \times \vec{F}_k| = \frac{i_a}{\underbrace{2}_{\text{Only half of the armature current flows in one conductor}}} \cdot \overbrace{l}^{\text{Axial length}} \cdot B_f \cdot \overbrace{r}^{\text{Rotor radius}}$$



TORQUE GENERATION

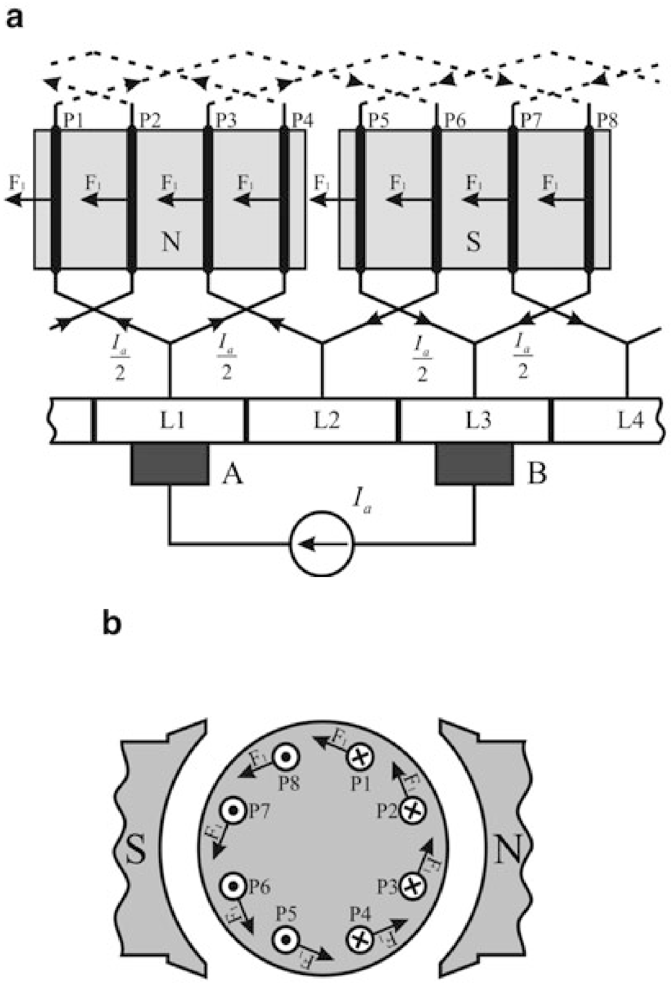
Electromagnetic torque is related to the excitation flux and armature current

- Each individual force contributes to the electromagnetic torque

$$T_{em,k} = |\vec{r}_k \times \vec{F}_k| = \frac{i_a}{2} \cdot l \cdot B_f \cdot r$$

- The total electromagnetic torque is the composition of the contribution of different conductors:

$$T_{em} = \sum_k T_{em,k} = 2\underbrace{\left(N_R \frac{W}{2\pi r}\right)}_{\text{Number of rotor conductors under one pole}} T_{em,k} = 2\left(N_R \frac{W}{2\pi r}\right)\frac{i_a}{2} \cdot l \cdot B_f \cdot r$$



(Note: the conductors under both poles contribute in the same way to the torque)

TORQUE GENERATION

Electromagnetic torque is related to the excitation flux and armature current

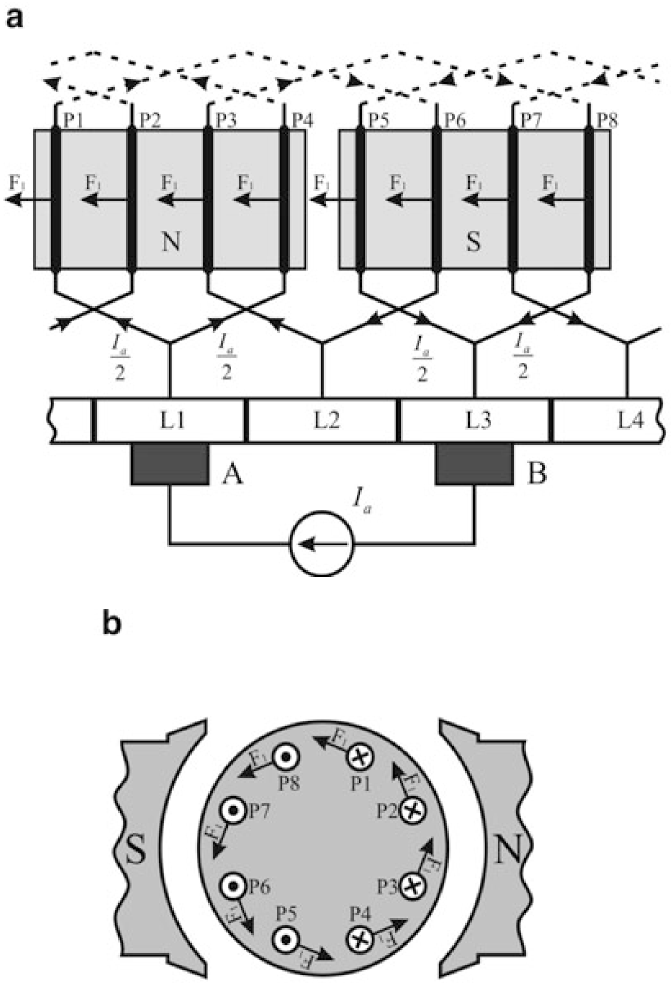
- Each individual force contributes to the electromagnetic torque

$$T_{em,k} = |\vec{r}_k \times \vec{F}_k| = \frac{i_a}{2} \cdot l \cdot B_f \cdot r$$

- The total electromagnetic torque is the composition of the contribution of different conductors:

$$T_{em} = \sum_k T_{em,k} = 2\left(N_R \, \frac{W}{2\pi r}\right) T_{em,k} = 2\left(N_R \, \frac{W}{2\pi r}\right) \frac{i_a}{2} \cdot l \cdot B_f \cdot r$$

$$= \left(\frac{N_R}{2\pi}\right) \cdot (l \, W \, B_f) \cdot i_a = \underbrace{k_m}_{\text{Torque Coefficient}} \Phi_f \, i_a$$

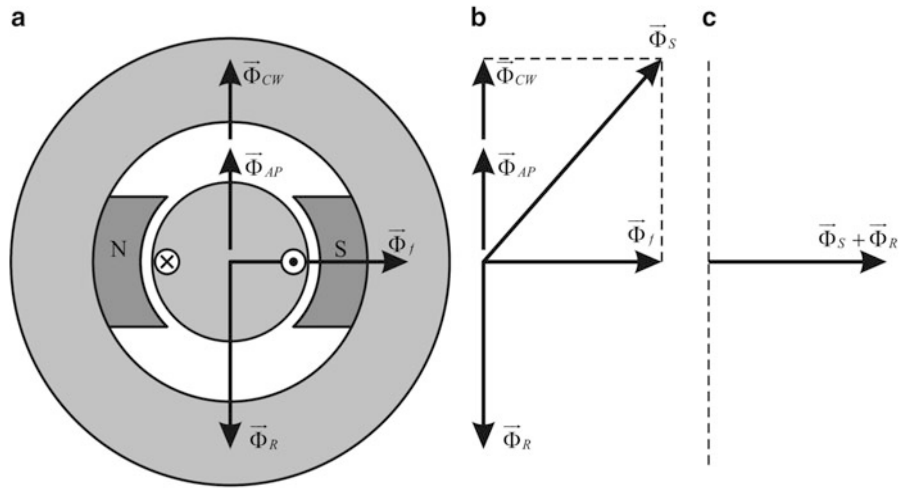


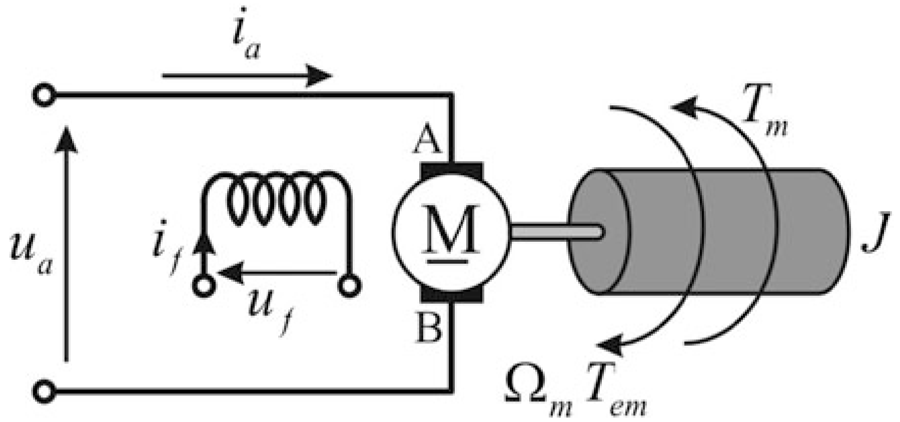
(Note: the conductors under both poles contribute in the same way to the torque)

DC MACHINE MODEL

Based on the theory presented so far, the mathematical model can be developed:

- The **excitation flux** is established along the **direct axis** either by excitation winding or permanent magnets
- The **armature (rotor) flux** is established along the **quadrature axis**
- the two axes are orthogonal and **the mutual inductance between them is zero** (assuming linear magnetic circuit)
- due to absence of interaction, transient phenomena of two axes are decoupled
- this makes the model of DC machine rather simple
- decoupled flux and torque are something very important for the high performance variable speed drives
- DC machines provide this by virtue of their construction





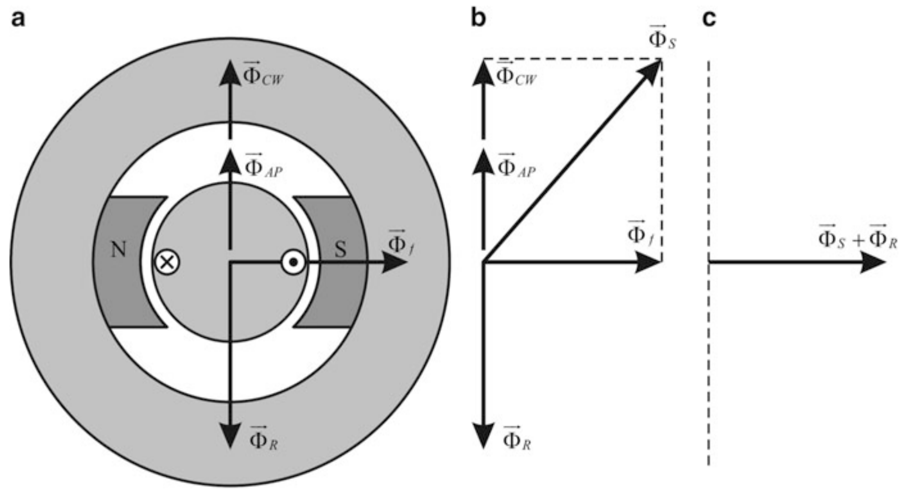
DC MACHINE MODEL

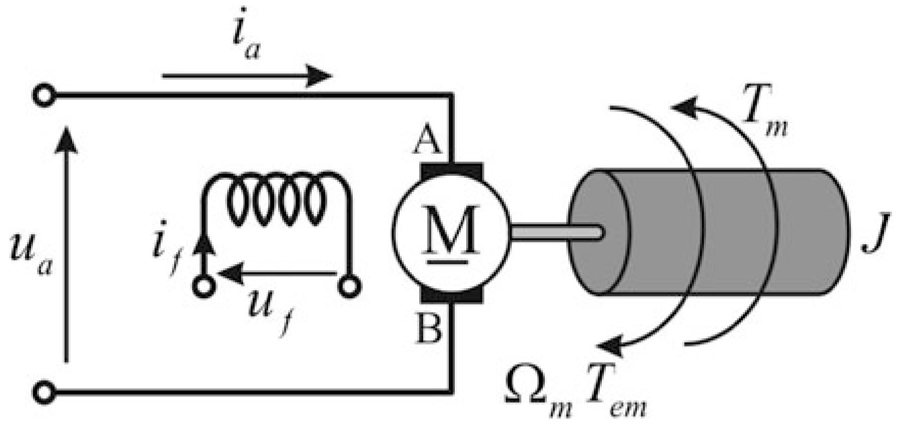
A separately excited DC machine is supplied from the two sources

- The stator (field) winding is supplied from the separate source u_f
- The rotor (armature) winding is supplied from the source u_a

The mathematical model includes:

- Two differential equations of voltage balance on the windings
- A differential equation describing the changes of angular speed (**Newton equation**)
- Algebraic relations between fluxes and currents (**inductance matrix**)
- An expression for the **electromagnetic torque**





VOLTAGE BALANCE EQUATION FOR THE EXCITATION

The MMF along the quadrature axis has no influence on the excitation flux

- The instantaneous value of the flux in the excitation winding is:

$$\Psi_f = N_f \cdot \Phi_f = L_f \cdot i_f$$

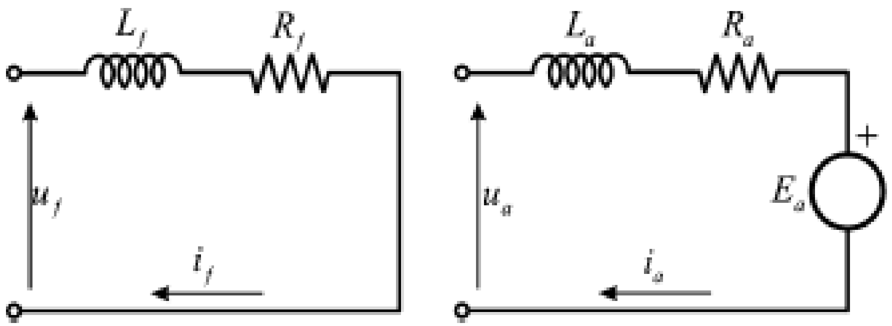
- The voltage balance equation for the excitation winding is:

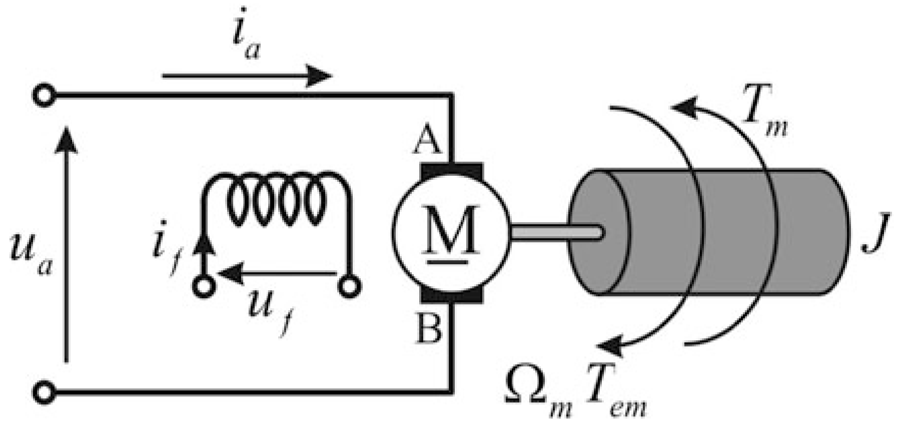
$$u_f = R_f\, i_f + \frac{\mathrm{d}\Psi_f}{\mathrm{d}t} = R_f\, i_f + L_f \frac{\mathrm{d}i_f}{\mathrm{d}t}$$

- The excitation winding can be simply represented as an **RL circuit** (first order system)

- The excitation flux is proportional to the excitation current

$$\Phi_f = \frac{\Psi_f}{N_f} = \frac{L_f}{N_f} \cdot i_f = L'_f i_f$$

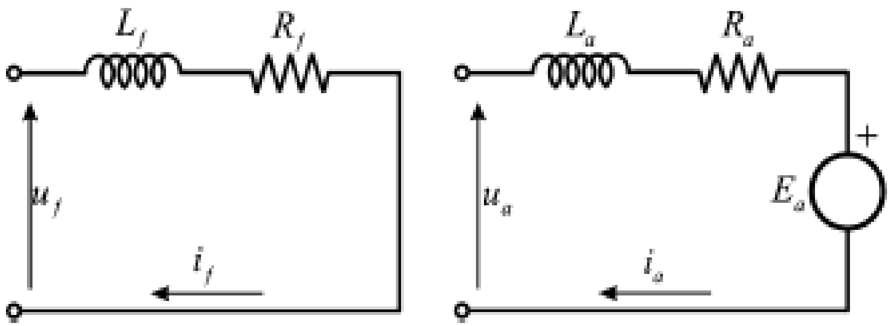




VOLTAGE BALANCE EQUATION FOR THE ARMATURE

The MMF along the quadrature axis has no influence on the excitation flux

- Rotor winding is also characterized by internal resistance and inductance
    - The magnetic resistance along the rotor path is relatively high due to the large air gap
    - The armature inductance is therefore two to three orders of magnitude lower than the excitation inductance

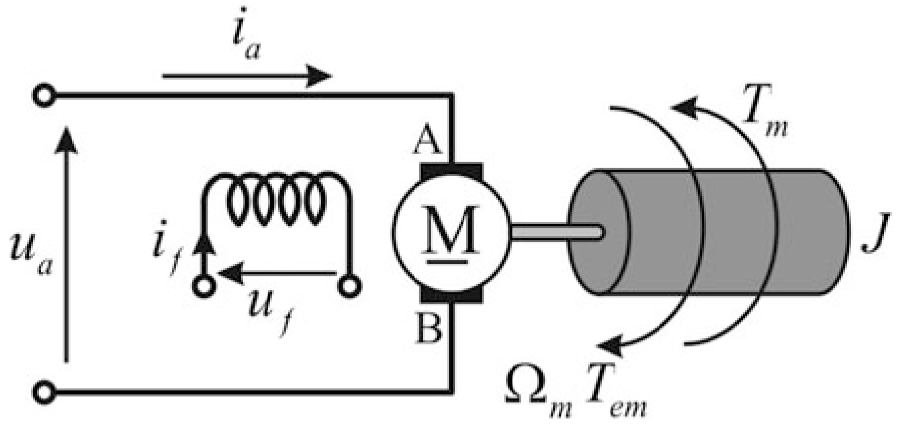


- In addition to resistance and inductance, rotor circuit has induced electromotive force.

$$u_a = R_a\, i_a + \frac{\mathrm{d}\Psi_a}{\mathrm{d}t} = R_a\, i_a + L_a \frac{\mathrm{d}i_a}{\mathrm{d}t} + E_a$$

$$= R_a\, i_a + L_a \frac{\mathrm{d}i_a}{\mathrm{d}t} + k_e \Phi_f \omega_m$$

- The armature winding can be simply represented as an **RL circuit** (first order system) **with a series voltage source** (proportional to the rotor speed)



MECHANICAL EQUATIONS

The mechanical subsystem can be modeled by Newton's equation

$$J \cdot \frac{\mathrm{d}\omega}{\mathrm{d}t} = T_{em} - T_m - k_F \cdot \omega$$

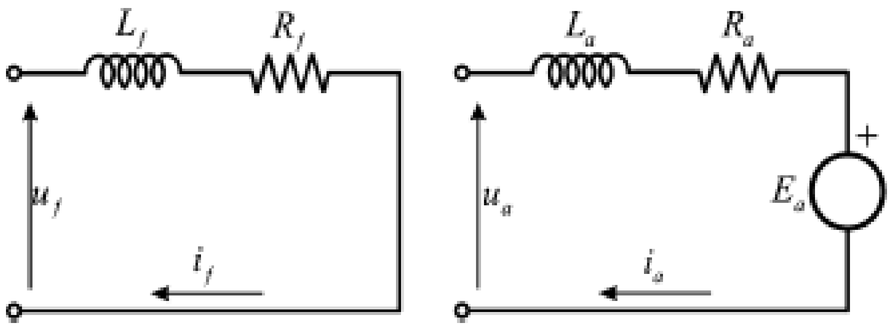
- The electromagnetic torque depends on the product of the excitation flux and the armature current

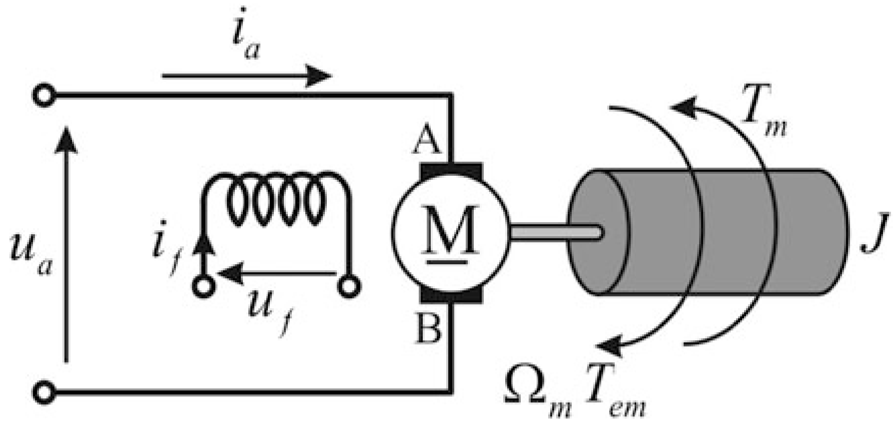
$$T_{em} = k_m \, \Phi_f \, i_a$$

- Note: the EMF constant and the Torque constant are the same (conservation of energy)

$$k_m = k_e$$

- The angle equation is not relevant for the machine model (the equivalent inductances are not depending on the position)





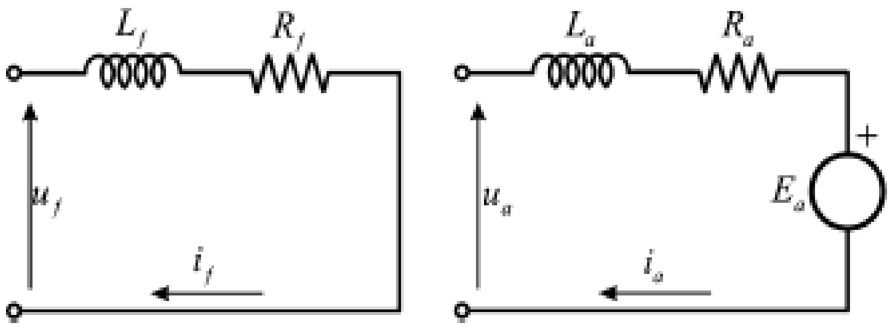
SUMMARY

Mathematical model of the DC machine

GENERAL DYNAMICAL MODEL OF A DC MACHINE

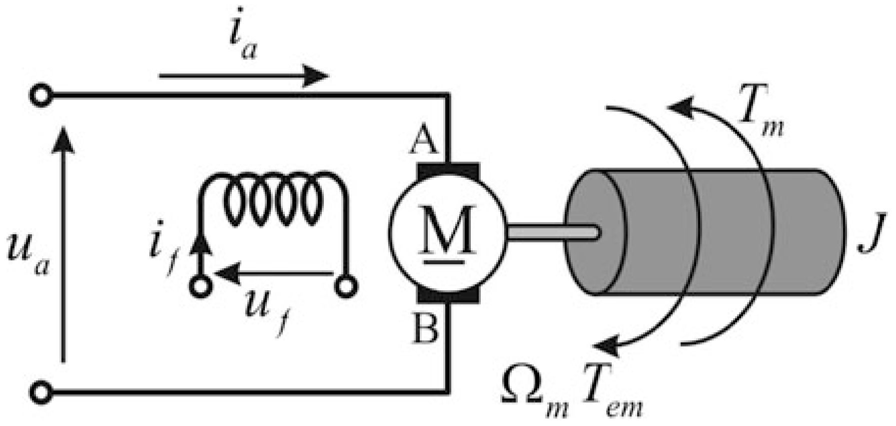
Electrical Equations

$$\begin{cases} u_a = R_a\, i_a + L_a \dfrac{\mathrm{d}i_a}{\mathrm{d}t} + E_a \\ u_f = R_f\, i_f + L_f \dfrac{\mathrm{d}i_f}{\mathrm{d}t} \\ E_a = k_e\, \Phi_f\, \omega_m \\ \Phi_f = L'_f\, i_f \end{cases}$$



Mechanical Equations

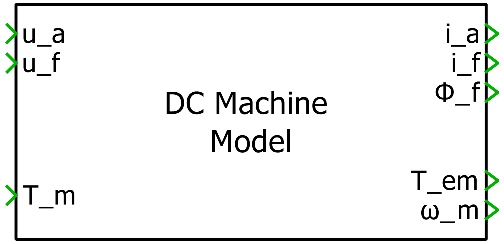
$$\begin{cases} J \cdot \dfrac{\mathrm{d}\omega_m}{\mathrm{d}t} = T_{em} - T_m - k_F \cdot \omega_m \\ T_{em} = k_m\, \Phi_f\, i_a \end{cases}$$



BLOCK DIAGRAM OF THE MODEL

With the help of Laplace transform block diagram of model is obtained

- The inputs into model are: excitation and armature voltage
- The state variables are: armature current, excitation current and rotor speed
- The outputs are: armature current, excitation flux, electromagnetic torque and the rotor speed



(To do in the Exercise Session)